

A METHOD FOR TRACKING TARGETS WITH HYPER-SPECTRAL DATA

TO ALL WHOM IT MAY CONCERN:

BE IT KNOWN THAT (1) MARCUS L. GRAHAM, (2) TOD E. LUGINBUHL, (3) ROY L. STREIT, and (4) MICHAEL J. WALSH, employees of the United States Government, citizens of the United States of America, and residents of (1) North Kingstown, County of Washington, State of Rhode Island, (2) Portsmouth, County of Newport, State of Rhode Island, (3) Portsmouth, County of Newport, State of Rhode Island, and (4) Somerset, County of Bristol, Commonwealth of Massachusetts have invented certain new and useful improvements entitled as set forth above of which the following is a specification:

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5 STATEMENT OF GOVERNMENT INTEREST

6 The invention described herein may be manufactured and used
7 by or for the Government of the United States of America for
8 governmental purposes without the payment of any royalties
9 thereon or therefore.

10

11 CROSS REFERENCE TO OTHER PATENT APPLICATIONS

12 Not applicable.

13

14 BACKGROUND OF THE INVENTION

15 (1) Field of the Invention

16 The present invention relates to remote sensing and remote
17 imaging, and more specifically to a method for processing hyper-
18 spectral remote sensor data for the purpose of displaying the
19 spatial tracks of energy sources in multi-spectral images
20 corresponding to the sensor data.

21 (2) Description of the Prior Art

22 Remote sensing of the energy signals of a moving vehicle or
23 energy source for the purposes of tracking the vehicle or energy
24 source has often been accomplished by measuring the intensity of

1 the energy signals with sensors specifically designed to detect
2 energy intensity. In remote sensing applications the sensor is
3 often a planar array of sensing cells, each cell responding to
4 the energy incident on its corresponding section of the array
5 surface. In other applications, such as acoustic sensing, the
6 received energy on sensor elements must be interpreted through a
7 beam forming function to yield energy intensity in a set of
8 spatially directed cells (more commonly called beams). Such a
9 method is designed to track energy peaks as they move over time
10 on the given set of sensor cells. The total broadband energy is
11 plotted and visually displayed. Targets appear as peaks of
12 energy in the display, and are tracked. One method of target
13 tracking based on sensor level data is the Histogram
14 Probabilistic Multi-Hypothesis Tracking (H-PMHT) algorithm. It
15 is an application of the Expectation-Maximization (EM) method of
16 target tracking. It uses a synthetic (multi-dimensional)
17 histogram interpretation of the received power levels in all of
18 the sensor cells. The data for the H-PMHT algorithm usually
19 consists of broadband intensities on a set of spatial sensor
20 cells. The H-PMHT algorithm has its limitations. For example,
21 in situations where more than one target is being tracked and
22 the targets cross paths, the intensity of the energy signals of
23 the targets merge, making it impossible to distinguish the
24 energy between the two targets. In such a situation, the

1 targets must be reacquired by the sensors after they have
2 crossed resulting in a gap and delay in tracking information.

3 U.S. Patent Application 10/214551 to Struzinski teaches a
4 method and system for predicting and detecting the crossing of
5 two target tracks in a bearing versus time coordinate frame.
6 The method/system uses a series of periodic bearing measurements
7 of the two target tracks to determine a bearing rate and a
8 projected intercept with a bearing axis of the bearing versus
9 time coordinate frame. A crossing time t_c for the two target
10 tracks is determined using the tracks' bearing rates and
11 projected intercepts. A prediction that the two target tracks
12 will cross results if a first inequality is satisfied while a
13 detection that the two target tracks have crossed results if a
14 second inequality is satisfied. This method does not, however,
15 address the problem of distinguishing between and identifying
16 the targets before, during and after they have crossed.

17 There is currently no reliable method by which targets can
18 be consistently tracked and distinguished as they cross paths.
19 What is needed is a method for tracking targets that does not
20 rely solely on broadband energy signal intensity, but also
21 utilizes the spectral aspects of the energy signal, combining
22 both intensity and spectral data so that crossing targets can be
23 tracked provided they have some degree of spectral distinction.

SUMMARY OF THE INVENTION

2 It is a general purpose and object of the present invention
3 to provide a method for tracking both the spatial sensor data
4 and hyper-spectral sensor data associated with a target.

5 It is a further purpose to estimate a frequency spectrum
6 for a target contribution that consists of only the target's
7 energy contribution as opposed to the target energy and the
8 noise energy together.

9 These objects are accomplished with the present invention
10 by taking the histogram model used in H-PMHT and extending it to
11 treat the problem of tracking using hyper-spectral data. In the
12 present invention each measurement scan is now a multi-
13 dimensional array wherein each spatial cell has an associated
14 vector of amplitudes in several (possibly disjoint) spectral
15 cells. The intensity data in the multi-dimensional array is
16 interpreted as the spatial-spectral histogram of a synthetic
17 shot process. A statistical model of the random variation of
18 individual cell intensities from scan to scan is required. The
19 procedure adopted in H-PMHT is to quantize the data vector into
20 a "pseudo-histogram," and then use a multinomial distribution to
21 model the cell counts where the PMHT target mixture model
22 parameterizes the multinomial distribution. The target mixture
23 model determines the cell probabilities that correspond to
24 expected cell counts.

1 The present invention modifies H-PMHT by using a non-
2 parametric spectral characterization of the energy intensity of
3 the target that is assumed known. The use of such a spectral
4 template enhances low signal to noise ratio (SNR) tracking and
5 allows discrimination of spectrally distinct sources as they
6 cross in the spatial domain. The track solutions from previous
7 batches are used to estimate the (non-parametric) spectral
8 characterization that is used to initiate the generation of the
9 updated solutions as new data is received and processed.

10 The present invention provides a mechanism for separating
11 the observed hyperspectral energy into the hyperspectral energy
12 for each source (including noise) using the known spectral
13 characteristics for each source. Completely general spectral
14 density functions are handled via the use of non-parametric
15 methods. In the alternative, the source spectrum is estimated
16 in a non-parametric fashion based on an initial track, allowing
17 the algorithm to adapt to the source spectrum in situ.

19 BRIEF DESCRIPTION OF THE DRAWINGS

20 A more complete understanding of the invention and many of
21 the attendant advantages thereto will be readily appreciated as
22 the same becomes better understood by reference to the following
23 detailed description when considered in conjunction with the

1 accompanying drawings depicting an underwater application as the
2 preferred embodiment wherein:

3 FIG. 1 shows an underwater vehicle towing sensors;

4 FIG. 2 shows sensors detecting energy from different
5 sources;

6 FIG. 3 shows the data cube created after raw sensor data is
7 processed; and

8 FIG. 4 shows a flow chart of the method.

9

10 DESCRIPTION OF THE PREFERRED EMBODIMENT

11 Referring now to FIG. 1 there is shown an underwater
12 vehicle 10 towing an array of sensors 15 arranged on a cable 20.
13 The sensors 15 are of a type known by those skilled in the art
14 of signal processing such as hydrophones. The sensors 15 are
15 capable of detecting energy signals and their intensities from
16 different directions as illustrated in FIG. 2, which shows two
17 vessels labeled k_1 and k_2 and the energy signals 17 emanating
18 from the vessels. The sensor data from each sensor 15 is
19 transmitted along cable 20 to data processors (not shown) within
20 underwater vehicle 10. The data processors take the raw sensor
21 data and create a data cube 25 as illustrated in FIG. 3. Each
22 such data cube 25 is a collection of smaller cubes referred to
23 as sensor cells 30 which correspond to the processed sensor data
24 generated by the sensors 15. Each sensor cell 30 contains

1 spatial measurements along the x-axis 31, spectral measurements
2 along the y-axis 32 and time measurements along the t-axis 33.
3 The side of each sensor cell 30 contained in the (x,t) plane
4 corresponding to spatial measurement is referred to as a spatial
5 cell 36. The side of each sensor cell 30 contained in the (y,t)
6 plane corresponding to spectral measurement is referred to as a
7 spectral cell 38. The processing and arrangement of the raw
8 sensor data into a data cube 25 composed of multiple sensor
9 cells 30 that are further composed of spatial cells 36 and
10 spectral cells 38 is known by those skilled in the art of signal
11 processing and is achieved by what is often termed beamforming
12 followed by spectral analysis of the beam intensity data using
13 standard discrete Fourier transform (DFT) techniques known in
14 the art. A single layer of the data cube 25 is referred to as a
15 scan of the sensor space 35 as illustrated in FIG. 3.

16 In the preferred embodiment of the present invention, let
17 $C = \{C_1, \dots, C_s\}$, $s \geq 1$, denote the collection of all possible sensor
18 cells 30. It is assumed that $C_i \cap C_j = \emptyset$ for all i and j and
19 that $C_1 \cup \dots \cup C_s = R^{\dim(C)}$, where $\dim(C)$ denotes the dimension of the
20 sensor space 35. Furthermore, the sensor cells 30 $C = \{C_1, \dots, C_s\}$ are the Cartesian products of U disjoint spatial cells 36
21 $\{\mathcal{D}_1, \dots, \mathcal{D}_U\}$ and V disjoint spectral cells 38 $\{\mathcal{E}_1, \dots, \mathcal{E}_V\}$. This
22 particular choice of spatial 36 and spectral 38 cells is

1 application dependent, but they are intrinsically fixed. The
2 total number of sensor cells 30 in a scan of the sensor space 35
3 is $S=UV$, and every cell C_i can be written in the form

4 $C_i = D_i x \epsilon_j$

5 for some (unique) choice of the cells D_i and ϵ_j . Let $D=D_1 \cup$
6 $\cup D_U$ and $\epsilon=\epsilon_1 \cup \dots \cup \epsilon_V$. The sensor cells 30 from which
7 measurements are available may vary from scan to scan. The
8 sensor cells 30 displayed at time t , for the scan of the sensor
9 space 35, are the Cartesian product of the spatial cells 36
10 $\{D_1(t), \dots, D_{U(t)}(t)\}$ and the spectral cells 38 $\{E_1(t), \dots$
11 $\dots, E_{V(t)}(t)\}$, so that the $(i, j)^{th}$ sensor cell 30 is $C_{ij}(t) = D_i(t) \times$
12 $E_j(t)$, where $1 \leq U(t) \leq U$ and $1 \leq V(t) \leq V$. The remaining sensor
13 cells 30 in the scan of sensor space 35 are said to be
14 truncated, and no measurements are collected for these cells at
15 time t .

16 Let the scan of sensor space 35 at time t be denoted by

17 $Z_t = \{z_{t,1,1}, \dots, z_{t,U(t),V(t)}\}$,

18 where $z_{tij} \geq 0$ is the output of the sensor space 35 at time t in
19 cell $C_{ij}(t)$, $i=1, 2, \dots, U(t)$, $j=1, 2, \dots, V(t)$. Let $\hbar^2 > 0$ be a
20 specified quantization level, and let n_{tij} denote the quantized
21 value corresponding to the intensity z_{tij} in cell $C_{ij}(t)$, where

22 $n_{tij} = \left\lfloor \frac{z_{tij}}{\hbar^2} \right\rfloor$, (1)

1 and $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x .
2 The use of the quantized values $\{n_{tij}\}$ instead of the
3 measurements $\{z_{tij}\}$ is an intermediate step in the development.
4 After deriving the auxiliary function of the H-PMHT algorithm
5 using the synthetic counts $\{n_{tij}\}$, the measurements $\{z_{tij}\}$ are
6 recovered in the limit as $\hbar^2 \rightarrow 0$.

7 The "rectangular" spatial-spectral sensor cell structure
8 enables simplifications to the basic equations of H-PMHT. These
9 equations are restated here with the updated notation
10 corresponding to this new cell structure. The cell probability,
11 P_{ij} , for the $(i, j)^{\text{th}}$ cell 30 takes the form

$$12 \quad P_{ij}(X_t) = \int_{C_{ij}(t)} f(u, v|X_t) du, dv, \quad (2)$$

13 where the sample Probability Density Function (PDF) $f(u, v|X_t)$ is
14 defined over all $(u, v) \in \mathbb{R}^{\text{dim}D} \times \mathbb{R}^{\text{dim}E} = \mathbb{R}^{\text{dim}C}$, by the mixture
15 density

$$16 \quad f(u, v|X_t) = \sum_{k=0}^M \pi_{tk} G_k(u, v|x_{tk}) \quad (3)$$

17 and where π_{tk} is the component mixing proportion,
18 $X_t = \{x_{t0}, \dots, x_{tM}\}$ are the component spatial state parameters and
19 $G_k(u, v|x_{tk})$ is the component PDF corresponding to target k if $k \geq$
20 1 and to noise if $k = 0$. The expected sensor space measurement
21 \bar{z}_{tij} takes the form

1

$$2 \quad \bar{z}_{tij} = \begin{cases} z_{tij} & \begin{cases} 1 \leq i \leq U(t), \\ 1 \leq j \leq V(t), \end{cases} \\ \|z_t\| \frac{P_{ij}(X_t)}{P(X_t)} & \begin{cases} U(t) + 1 \leq i \leq U, \\ V(t) + 1 \leq j \leq V, \end{cases} \end{cases} \quad (4)$$

3

4

$$5 \quad \text{where } \|z_t\| = \sum_{i=1}^{U(t)} \sum_{j=1}^{V(t)} z_{tij},$$

$$6 \quad P(X_t) = \sum_{i=1}^{U(t)} \sum_{j=1}^{V(t)} P_{ij}(X_t), \quad (5)$$

7. and X_t' is the last estimate of X_t . Thus, from (4) it may be
 8. seen that expected measurements exist for all cells, even those
 9. truncated in the observation. After taking the quantization
 10. limit, $h^2 \rightarrow 0$, the H-PMHT auxiliary functions become

$$11 \quad Q_{t\pi} = \sum_{k=0}^M \left[\sum_{i=1}^U \sum_{j=1}^V \frac{\bar{z}_{tij}}{P_{ij}(X_t')} \int_{C_{ij}(t)} G_k(u, v | x_{tk}') du dv \right] \pi_{tk} \log \pi_{tk} \quad (6)$$

12 and

$$13 \quad Q_{kX} = \sum_{t=1}^T \frac{\|z_t\|}{P(X_t')} \log P_{\Xi_{tk} | \Xi_{t-1,k}}(x_{tk} | x_{t-1,k}) + \sum_{t=1}^T \sum_{i=1}^U \sum_{j=1}^V \frac{\pi_{tk} \bar{z}_{tij}}{P_{ij}(X_t')} \int_{C_{ij}(t)} G_k(u, v | x_{tk}') \log G_k(u, v | x_{tk}) du dv. \quad (7)$$

14 The density $P_{\Xi_{tk} | \Xi_{t-1,k}}(x_{tk} | x_{t-1,k})$ for $t=1, 2, \dots, T$ describes the
 15 Markov process for the state of target k .

1 Let the spectral PDF of target k be denoted by $S_k(v)$, so
 2 that

3
$$\int_{-\infty}^{\infty} S_k(v) dv = 1. \quad (8)$$

4 The spectral PDF is equal to the traditional power spectrum
 5 normalized so that its integral over ϵ is one. Because the target
 6 spatial and spectral characteristics are independent by
 7 assumption, each component $G_k(u, v | x_{tk})$ of the sample PDF factors:

8
$$G_k(u, v | x_{tk}) = g_k(u | x_{tk}) S_k(v) \quad (9)$$

9 where $g_k(u | x_{tk})$ is the spatial PDF of component k. Independence
 10 enables integrals over $C_{ij}(t)$ to be rewritten as products of
 11 integrals, so that

12
$$\int_{C_{ij}(t)} G_k(u, v | x_{tk}) du dv = \int_{E_j(t)} S_k(v) dv \int_{D_i(t)} g_k(u | x_{tk}) du. \quad (10)$$

13 and, using the mixture (3) and the definition (2),

14
$$P_{ij}(X_t) = \sum_{k=0}^M \pi_{tk} \int_{E_j(t)} S_k(v) dv \int_{D_i(t)} g_k(u | x_{tk}) du. \quad (11)$$

15 Substituting (10) into (6) gives

16
$$Q_{t\pi} = \sum_{k=0}^M \left[\sum_{i=1}^U \Psi_{tki} \int_{D_i(t)} g_k(u | x_{tk}) du \right] \pi_{tk} \log \pi_{tk}, \quad (12)$$

17 where

18
$$\Psi_{tki} = \left(\sum_{j=1}^V \frac{\bar{z}_{tij} \int_{E_j(t)} S_k(v) dv}{P_{ij}(X_t)} \right) \quad (13)$$

1 is analogous to a normalized matched filter output for target k
2 on spatial cell i at time t , and $P_{ij}(X_t)$ is given in (11).

3 Similarly, (7) becomes

4

$$Q_{kx} = \sum_{t=1}^T \frac{\|Z_t\|}{P(X_t)} \log p_{\Xi_{t,k}|\Xi_{t-1,k}}(x_{tk}|x_{t-1,k}) + \sum_{t=1}^T \pi_{tk} \sum_{i=1}^U \Psi_{tki} \\ \times \int_{D_i(t)} g_k(u|x_{tk}) \log g_k(u|x_{tk}) du. \quad (14)$$

5 There is an additional term in (14), but it is omitted here
6 because it depends on $x_{t,k}$ and not on $x_{t,k}$, and thus does not
7 influence the M-step of the EM method. It should be noted at
8 this point that it is not necessary to have an analytic
9 expression for $S_k(v)$ to utilize (12) and (14). It is sufficient
10 to know the values of the set of integrals $\left\{ \int_{E_j(t)} S_k(v) dv \right\}$,

11 $j = 1, \dots, V$, for each target k . This vector of spectral cell 38
12 probabilities is a non-parametric description of the target
13 spectral density sufficient for the problem at hand.

14 At this stage, specific parametric forms are adopted for
15 the target and measurement processes. For target k , $k=1, \dots, M$,
16 the process evolution is defined by

17 $p_{\Xi_{t,k}|\Xi_{t-1,k}}(x_{tk}|x_{t-1,k}) = N(x_{tk}; F_{t-1,k}x_{t-1,k}, Q_{t-1,k}) \quad (15)$

18 where $N(x; \mu, \Sigma)$ is the multivariate normal distribution in x with
19 mean μ and covariance Σ . The measurements are characterized by

$$g_k(u|x_{tk}) = N(u; H_{tk}x_{tk}, R_{tk}). \quad (16)$$

2 The covariance matrix R_{tk} relates to the spatial extent, or
 3 spreading, of the energy about its expected location given by
 4 $H_{tk}x_{tk}$. Estimates of $\{\hat{\pi}_{tk}\}$ are obtained using a Lagrange
 5 multiplier technique. The result is

$$\hat{\pi}_{tk} = \frac{\pi_{tk}}{\lambda_t} \sum_{i=1}^U \Psi_{tki} \int_{D_i(t)} N(u; H_{tk}x_{tk}^i, R_{tk}) du, \quad (17)$$

7 where

$$\lambda_t = \sum_{k=0}^M \pi_{tk} \left[\sum_{i=1}^U \Psi_{tki} \int_{D_i(t)} \mathcal{N}(u; H_{tk}x_{tk}, R_{tk}) du \right] = \sum_{i=1}^U \sum_{j=1}^V \bar{z}_{tij} \quad (18)$$

9 The last form follows by taking the sum over k innermost and
 10 using Eq. (11).

11 Estimates for the state variables are obtained by setting
12 the gradient of the auxiliary function Q_{kx} to zero and solving;
13 however, as in the earlier developments of H-PMHT, an
14 alternative approach is taken because it exploits the Kalman
15 filter as an efficient computational algorithm. The details of
16 the Kalman filter steps are omitted here, however, the synthetic
17 spatial measurements used in the filter for target k now have
18 the form

$$z_{tk} = \frac{1}{v_{tk}} \sum_{i=1}^U \Psi_{tki} \int_{D_i(t)} u N(u; H_{tk} x_{tk}^i, R_{tk}) du, \quad (19)$$

20 where

1

$$v_{tk} = \sum_{i=1}^U \Psi_{tki} \int_{D_i(t)} N(u; H_{tk}x_{tk}, R_{tk}) du. \quad (20)$$

2 The synthetic process and measurement noise covariance matrices
 3 used in conjunction with this synthetic measurement are
 4 respectively given by

5

$$\tilde{Q}_{tk} = \frac{P(X'_{t+1})}{\|Z_{t+1}\|} Q_{tk}, \quad 0 \leq t \leq T-1 \quad (21)$$

6 and

7

$$\tilde{R}_{tk} = \frac{R_{tk}}{\pi_{tk} v_{tk}}, \quad 1 \leq t \leq T \quad (22)$$

8 Let $\{\pi_{tk}^I\}$ be the set of estimated mixing proportions and $\{x_{tk}^I\}$
 9 and $\{R_{tk}^I\}$ define the signal states and width parameters at the I -
 10 th EM iteration. For simplicity and robustness, assume that
 11 $\{\pi_{tk}^I\} = \{\pi_k^I\}$ and $\{R_{tk}^I\} = \{R_k^I\}$ for all $t=1, \dots, T$ in the batch of scans
 12 of the sensor space 35. These restrictions, tantamount to
 13 statistical stationarity, are most often reasonable over the
 14 data intervals of interest. Further, since the spectral density
 15 is never itself required, we will denote the needed integrals by

16

$$S_{kj} = \int_{E_j(t)} S_k(v) dv.$$

17 The method as described below is illustrated in the flow
 18 chart in FIG. 4. At the beginning of the method (the 0-th
 19 iteration), the mixing proportions $\{\pi_k^{(0)}\}$ are initialized so that

1 $\pi_k^{(0)} > 0$ and $\pi_0^{(0)} + \pi_1^{(0)} + \dots + \pi_M^{(0)} = 1$. The signal state sequences
2 $x_k^{(0)} = \{x_{1k}^{(0)}, \dots, x_{tk}^{(0)}, \dots, x_{Tk}^{(0)}\}$ are initialized with nominal values
3 for $k=1, \dots, M$, and the signal widths $\{R_1^{(0)}, R_2^{(0)}, \dots, R_M^{(0)}\}$ are set
4 nominally above the expected signal widths so that the tracks
5 are better able to "see" nearby energy when poorly initialized.
6 The simple case of $x_k^{(0)} = \{x_{0,k}^{(0)}, \dots, x_{0,k}^{(0)}, \dots, x_{0,k}^{(0)}\}$, (stationary
7 target), has proven an effective starting point in many cases.

8 The process covariance matrices $Q_t = \{Q_{t,1}, Q_{t,2}, \dots, Q_{t,M}\}$ are
9 initialized with values tailored to the problem at hand so as to
10 compromise between smooth tracking and the ability to follow
11 through aberrant behavior. Typically it is assumed that the
12 process covariance matrices are constant over time
13 $Q_t = Q = \{Q_1, Q_2, \dots, Q_M\}$. In order to get the iterative estimator
14 started, initial values are also required for the target state
15 spectral distributions $S = \{S_1, S_2, \dots, S_M\}$. The simple case of

16 $S_k = \left\{ \frac{1}{V}, \frac{1}{V}, \dots, \frac{1}{V} \right\}$ has proven an effective starting point for
17 estimating the spectra of spatially isolated targets. The above
18 described initialization of target parameters is step 50 in FIG.
19 4.

20 For iterations $l=1, 2, \dots$, the following quantities are
21 computed:

1 1. Component spatial cell probabilities for $t=1, \dots, T$,
 2 $i=1, \dots, U$, and $k=0, 1, \dots, M$:

$$4 \quad P_{ki}^{(l)}(x_t) = \begin{cases} 1/U, & k = 0 \\ \int_{D_i} N(\tau; H_{tk} x_{tk}^{(l-1)}, R_{tk}^{(l-1)}) d\tau, & k \neq 0 \end{cases} \quad (23)$$

5
 6 2. Component spatial/spectral cell probabilities for
 7 $t=1, \dots, T$ and $i=1, \dots, U$, $j=1, \dots, V$, and $k=0, 1, \dots$
 8 $, M$:

$$9 \quad P_{kij}^{(l)}(x_t) = P_{ki}^{(l)}(x_t) S_{kj}. \quad (24)$$

10 3. Total spatial/spectral cell probabilities for $t=1, \dots, T$ and $i=1, \dots, U$, $j=1, \dots, V$:

$$12 \quad P_{ij}^{(l)}(x_t) = \sum_{k=0}^M \pi_{tk}^{(l-1)} P_{kij}^{(l)}(x_t). \quad (25)$$

13 4. Total scan probabilities for $t=1, \dots, T$:

$$14 \quad P_{(X_t)}^{(l)} = \sum_{i=1}^U \sum_{j=1}^V P_{ij}^{(l)}(x_t). \quad (26)$$

15 5. Expected sensor space measurement \bar{z}_{tij} for $t=1, \dots, T$
 16 $i=1, \dots, U$, and $j=1, \dots, V$ using equation (4),

17
 18 6. Spatial cell first moments for $t=1, \dots, T$ $i=1, \dots, U$, and $k=1, \dots, M$:

$$20 \quad \mu_{tki}^{(l)} = \int_{D_i} \tau N(\tau; H_{tk} x_{tk}^{(l-1)}, R_{tk}^{(l-1)}) d\tau. \quad (27)$$

1 7. Relative mode contributions for $t=1, \dots, T$ and
 2 $k=0, 1, \dots, M$:

$$3 \quad v_{tk} = \sum_{i=1}^U \sum_{j=1}^V \frac{z_{tij} p_{kij}^{(l)}(x_t)}{p_{ij}^{(l)}(x_t)} \quad (28)$$

4 8. Estimated mixing proportions for $t=1, \dots, T$ and
 5 $k=0, 1, \dots, M$:

$$6 \quad \pi_{tk}^{(l)} = \frac{\pi_{tk}^{(l-1)} v_{tk}}{\sum_{k'=0}^M \pi_{tk'}^{(l-1)} v_{tk}} \quad (29)$$

7
 8 9. Synthetic measurements for $t=1, \dots, T$ and $k=1, \dots, M$:

$$10 \quad \tilde{z}_{tk}^{(l)} = \frac{1}{v_{tk}} \sum_{i=1}^U \sum_{j=1}^V \frac{z_{tij} s_{kj} \mu_{tki}^{(l)}}{p_{ij}^{(l)}(x_t)} \quad (30)$$

11 10. Synthetic measurement covariance matrices for $t=1, \dots, T$ and $k=1, \dots, M$:

$$13 \quad \tilde{R}_{tk}^{(l)} = \frac{R_{tk}^{(l-1)}}{\pi_{tk}^{(l-1)} v_{tk}} \quad (31)$$

14 11. Synthetic process covariance matrices for $t=0, 1, \dots, T-1$ and $k=1, \dots, M$:

$$16 \quad \tilde{Q}_{tk}^{(l)} = \frac{p_{tk}^{(l)}(x_t)}{\|z_t\|} Q_{tk} \quad (32)$$

17 where Q_{tk} is treated as a control parameter for the
 18 process description, and most commonly $Q_{tk}=Q_k$ for all
 19 $t=1, \dots, T$ in the batch.

12. Estimated spatial states 55 in FIG.4

2 $x^{(l)} = \{x_{01}^{(l)}, \dots, x_{tk}^{(l)}, \dots, x_{TM}^{(l)}\}$ for $t=0, 1, \dots, T$ and $k=1, \dots$

3 $, M$, using (for computational efficiency) a recursive

4 Kalman smoothing filter, on the synthetic data $\tilde{z}_{tk}^{(l)}$ with
5 process and measurement matrices corresponding to

6 $F_{tk}, \tilde{Q}_{tk}^{(l)}, H_{tk}, \tilde{R}_{tk}^{(l)}$.

7 13. Spatial cell second moments for $t=1, \dots, T, i=1, \dots$
8 $, U$. and $k=1, \dots, M$:

9 $\sigma_{tki}^{(l)} = \int_{D_i} \left(\tau - H_{tk} x_{tk}^{(l-1)} \right)^2 N(\tau; H_{tk} x_{tk}^{(l-1)}, R_{tk}^{(l-1)}) d\tau. \quad (33)$

10 14. Average signal width estimates 60 in FIG.4 for $k=1, \dots$
11 $, M$:

12 $R_k^{(l)} = \left(\frac{1}{\sum_{t=1}^T v_{tk}} \right) \sum_{t=1}^T \sum_{i=1}^U \sum_{j=1}^V \frac{z_{tij} s_{kj} \sigma_{tki}^{(l)}}{P_{ij}^{(l)}(x_t)} \quad (34)$

13 At the completion of iterations l the estimated signal states $x^{(l)}$
14 and their width estimates $R^{(l)} = \{R_1^{(l)}, R_2^{(l)}, \dots, R_M^{(l)}\}$ constitute
15 the track estimate output.

16 15. Using the track estimate output, compute the average
17 synthetic spectral power 65 in FIG. 4 for $j=1, \dots, V$,
18 and $k=1, \dots, M$:

1

$$\hat{s}_{kj} = \left(\frac{1}{T}\right) \sum_{t=1}^T \frac{1}{v_{tk}} \sum_{i=1}^U \frac{z_{tij} p_{kij}^{(l+1)}(x_t)}{p_{ij}^{(l+1)}(x_t)} \quad (35)$$

2 (Note from (35) that $\sum_{j=1}^V s_{kj} \equiv 1.$)

3 The resulting combination of processed spatial, signal
4 width and spectral estimates are linked chronologically 70 and
5 displayed as an image on a computer display screen 75 using
6 display methods known in the art.

7 The advantages of the present invention over the prior art
8 are that the resulting method has improved crossing track
9 performance on sources that have some degree of spectral
10 distinction. The present invention also avoids the need for
11 thresholding and peak-picking to produce point measurements.

12 The spectral estimates (35) may be used to initiate this
13 estimator when run on subsequent batches of data. In the
14 preferred embodiment as a new scan is received the oldest scan
15 of the batch is dropped and the estimation method including the
16 steps 1 through 14 (formulae (23) through (35)) as stated above
17 is run in "sliding batch" fashion using the batch length that
18 provides sufficient smoothing without being unnecessarily long.
19 In this case, t in the equations represents the time index
20 within the batch under consideration. The track and spectral
21 estimates from the previous batch are used as initial values to
22 start the iterations as outlined.

1 The target specific spectral estimates (35) constitute
2 outputs unto themselves and can be easily computed for arbitrary
3 track sequences x' and R' used in place of $x^{(l)}$ and $R^{(l)}$. The
4 resulting spectral estimates have been termed "track conditioned
5 spectral estimates," and they serve to give a spectral
6 characterization to tracks generated via other means.

7 Obviously many modifications and variations of the present
8 invention may become apparent in light of the above teachings.
9 For example: $g_k(u/x_{tk})$ may take a parametric form other than the
10 normal density given in (16), $g_0(u)$ may be other than the uniform
11 density as implied by (23). While it was shown here that the
12 spectrum could be handled in a non-parametric form, the methods
13 are readily extended to treat a parametric spectral description.

14 In light of the above, it is therefore understood that
15 within the scope of the appended claims, the invention may be
16 practiced otherwise than as specifically described.